# STATUS AND PROSPECTS FOR LATTICE CALCULATIONS IN HEAVY QUARK PHYSICS*

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ABSTRACT

The current status of lattice calculation of weak matrix elements for heavy quark systems is reviewed. After an assessment of systematic errors in present simulations, results for the $B$ meson decay constant, the $B$ parameter $B_B$ and semi-leptonic heavy-to-light and heavy-to-heavy transitions are discussed. The final topic are lattice results for heavy baryon spectroscopy.

## 1. Introduction: Lattice Approach to Heavy Quark Systems

Heavy quark systems play an important rôle in the study of the less well-known elements of the CKM matrix, which are central to our understanding of the origin of CP violation, and may also contain information of physics beyond the Standard Model. Set against this background, the lattice formulation of QCD provides a framework for the calculation of hadron masses and weak matrix elements from first principles. Since the lattice approach is intrinsically non-perturbative, one may tackle the large theoretical uncertainties due to the strong interaction in weak decay amplitudes.

Lattice QCD replaces space-time by a four-dimensional, euclidean, hypercubic lattice of size $L^3 \cdot T$. The sites are separated by the lattice spacing $a$, which acts as an UV cut-off. One problem encountered in current simulations is that typical values of $a^{-1}$ lie in the range 2–3.5 GeV. Therefore one expects that discretisation errors ("lattice artefacts") will distort the results already for charm physics. Also, $b$ quarks cannot be studied directly, since their mass is above the UV cut-off.

Several methods are being used to circumvent this problem. It is now customary for simulations in heavy quark physics to cancel the leading discretisation error by employing so-called *improved* actions and operators [1], or by absorbing it into a rescaling of the quark fields [2]. Quantities can then be computed around $m_{\rm charm}$ and extrapolated to the $b$ quark mass. Alternatively, one can use the *static approximation* and perform the simulation at infinitely heavy quark mass, using the leading term of the expansion of the heavy quark propagator in $1/m_Q$. Finally, one can employ a *non-relativistic* formulation of the QCD Lagrangian. It is obvious that none of these methods is entirely satisfactory, but that they provide complementary information about heavy quark systems.

Apart from lattice artefacts, the main systematic errors in lattice simulations include the effects due to neglecting internal quark loops by using the so-called *quenched approximation.* The normalisation of lattice operators is another source of systematic uncertainties; due to the explicit breaking of chiral symmetry by the regularisation proce-

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dure, lattice operators are in general related to their continuum counterparts via (finite) normalisation constants, whose numerical values are usually not known very precisely. Also, the explicit breaking of chiral symmetry and Lorentz invariance leads to mixing with higher dimension operators. Hence, the connection between the matrix element of a continuum operator $\widehat{O}$ and its lattice counterparts is in general given by

$$\langle\widehat{O}\rangle^{\rm cont} = \sum_i Z_i(g^2)\langle\widehat{O}_i\rangle^{\rm latt} + \mathcal{O}(a), \tag{1}$$

where the $Z_i$'s are the appropriate normalisation and matching factors.

Finally, lattice estimates of dimensionful quantities are subject to uncertainties in the lattice scale. They arise from the fact that different quantities which are used to set the scale $a^{-1}$ [GeV] give different results. This is closely related to using the quenched approximation, since loop effects are not expected to be the same for different quantities.

## 2. Leptonic $B$ Decays and $B^0 - \overline{B^0}$ Mixing

The decay constants of heavy-light pseudoscalar and vector mesons, $f_P$ and $f_V$, are related to the matrix elements of the lattice axial and vector currents trough

$$\langle 0|A_4^{\rm latt}(0)|P\rangle \sim M_P\, f_P/Z_A, \qquad \langle 0|V_j^{\rm latt}(0)|V\rangle \sim \epsilon_j\, M_V^2 (f_V Z_V)^{-1}, \tag{2}$$

where $Z_A$, $Z_V$ are the normalisation constants of the currents. The matrix elements as well as the pseudoscalar and vector masses $M_P$, $M_V$ are extracted from mesonic two-point correlation functions through a fitting procedure. HQET predicts scaling laws for the combination $f_P\sqrt{M_P}$, which is expected to behave like a constant as $M_P \to \infty$. Furthermore, in the infinite mass limit, the HQ-spin symmetry predicts that pseudoscalar and vector decay constants become degenerate (up to short-distance corrections). Hence

$$\widetilde{U}(M) \equiv \frac{f_V f_P}{M} \Big/ \left\{1 + \tfrac{2}{3\pi}\alpha_s(M)\right\} = 1 + O(1/M) \tag{3}$$

It has been shown that $\widetilde{U}(M)$ indeed approaches one in the infinite mass limit [3]. However, lattice studies have revealed that the $1/M$ corrections to the scaling laws are large; for the decay constant they amount to about 15% at the $B$ mass and about 40% at the mass of the $D$ meson.

In Table 1 we show the results for the decay constants $f_D$, $f_{D_s}$ and $f_B$ from various lattice calculations using relativistic heavy quarks.[†] Expressing the results for the decay constant at the individual values of $a$ in terms of a common hadronic scale $r_0 \simeq 0.5\,\mathrm{fm}$ [11], one can extrapolate the dimensionless quantity $f_P r_0$ to the continuum limit, $a \to 0$. Repeating the procedure using lattice data for $f_\pi$, one obtains the continuum result from

$$f_P\,[\mathrm{MeV}] = 132\,\mathrm{MeV}\left(f_P r_0/f_\pi r_0\right)\Big|_{a=0}. \tag{4}$$

The results from this analysis using relativistic heavy quarks read

$$\begin{aligned} f_D &= 205 \pm 45\,\mathrm{MeV}, \qquad & f_{D_s} &= 220 \pm 50\,\mathrm{MeV} \\ f_B &= 170\ {}^{+55}_{-50}\,\mathrm{MeV}, \qquad & f_{B_s}/f_B &= 1.13 \pm 0.14. \end{aligned} \tag{5}$$

Table 1: Lattice estimates for the pseudoscalar decay constants from different collaborations. All data were obtained in the quenched approximation using relativistic heavy quarks.

| Collab. | $a$ [fm] | $f_D$ [MeV] | $f_{D_s}$ [MeV] | $f_B$ [MeV] | $f_{B_s}/f_B$ |
|---|---|---|---|---|---|
| MILC [4] | 0 | 182(3)(9)(22) | 198(5)(10)(19) | 151(5)(16)(26) | 1.11(2)(4)(8) |
| BLS [5] | 0.059 | 208(9)(35)(12) | 230(5)(10)(19) | 187(10)(34)(15) | 1.11(6) |
| LANL [6] | 0.083 | $229(7)\ {}^{+20}_{-16}$ | $260(4)\ {}^{+27}_{-22}$ | | |
| | 0 | 186(29) | 218(15) | | |
| UKQCD [3] | 0.069 | $185\ {}^{+4}_{-3}\ {}^{+42}_{-\ 7}$ | $212(4)\ {}^{+46}_{-\ 7}$ | $160(6)\ {}^{+59}_{-19}$ | $1.22\ {}^{+4}_{-3}$ |
| | 0.083 | $199(15)\ {}^{+27}_{-19}$ | $225(15)\ {}^{+30}_{-22}$ | $176(25)\ {}^{+33}_{-15}$ | 1.17(12) |
| ELC [7] | 0.051 | 210(40) | 230(50) | 205(40) | |
| JLQCD [8] | 0.059 | 216(17) | 240(17) | 182(16) | |
| | 0.078 | 206(12) | 237(14) | 192(11) | |
| PCW [10] | 0 | 170(30) | | 180(50) | 1.09(2)(5) |
| PCW [9] | 0.083 | 198(17) | 209(18) | | |

The $B$ meson decay constant together with the $B$ parameter $B_B$ is of great importance for the study of $B^0 - \overline{B^0}$ mixing. The renormalisation group invariant $B$ parameter $B_B$ is defined via $B_B = \alpha_s(\mu)^{-2/\beta_0}\,\langle\overline{B^0}\,|\,O_L(\mu)\,|\,B^0\rangle/\frac{8}{3}\,f_B^2\,M_B^2$, where $O_L(\mu)$ is the $\Delta B = 2$ four fermion operator. In a recent study, estimates for the $B$ parameter using the static approximation were obtained [12]

$$B_{B_d} = 1.02\ {}^{+5}_{-6}\ {}^{+3}_{-2}, \qquad B_{B_s} = 1.04\ {}^{+4}_{-5}\ {}^{+2}_{-1}, \tag{6}$$

where the first error is statistical, and the second is an estimate of systematic errors. The authors attribute a further systematic error of 15–20% to the uncertainty in the perturbative matching factors. The above result indicates that SU(3)-flavour breaking effects are small for $B_B$, whereas they can be quite sizeable in the case of $f_B$.

These results can be applied to the ratio of $B^0 - \overline{B^0}$ mixing parameters $x_s/x_d$

$$\frac{x_s}{x_d} = \frac{\tau_{B_s}}{\tau_{B_d}}\,\frac{\hat{\eta}_{B_s}}{\hat{\eta}_{B_d}}\,\frac{M_{B_s}}{M_{B_d}}\,\frac{f_{B_s}^2\,B_{B_s}}{f_{B_d}^2\,B_{B_d}}\,\frac{|V_{ts}|^2}{|V_{td}|^2} = (1.37 \pm 0.39)\,\frac{|V_{ts}|^2}{|V_{td}|^2}. \tag{7}$$

In conjunction with the experimental value $x_d = 0.71(6)$ [13], the above results can also be used to predict $x_s$, provided $\frac{|V_{ts}|^2}{|V_{td}|^2}$ is constrained using global fits [14]. Choosing $B_K = 0.8 \pm 0.2$, and taking our lattice estimate $f_B = 170 \pm 55\,\mathrm{MeV}$ one obtains

$$x_s = 13.4 \pm 4.0\ {}^{+10.3}_{-\ 3.7}\ {}^{+1.3}_{-0.7}, \tag{8}$$

where the first error mainly reflects the error in the ratio $f_{B_s}/f_B$, the second is due to the uncertainty in the actual value of $f_B$, and the third arises from the uncertainty in $B_K$. Clearly, much more precise values of $f_B$ are needed in order to predict $x_s$ more reliably.

[†]We use a convention in which $f_\pi = 132\,\mathrm{MeV}$.

Table 2: Lattice results for form factors relevant for semi-leptonic $B \to \pi\ell\overline{\nu}_\ell$, $\rho\ell\overline{\nu}_\ell$ decays. All results are obtained using propagating heavy quarks with the leading discretisation errors subtracted, except for ELC.

| Collab. | $a$ [fm] | $f_+(0)$ | $V(0)$ | $A_1(0)$ | $A_2(0)$ |
|---|---|---|---|---|---|
| ELC [18] | 0.051 | 0.30(14)(5) | 0.37(11) | 0.22(5) | 0.49(21)(5) |
| APE [19] | 0.083 | 0.35(8) | 0.53(31) | 0.24(12) | 0.27(80) |
| UKQCD [20,21] | 0.069 | 0.23(2) | | $0.27\ {}^{+7}_{-4}\ {}^{+3}_{-3}$ | $0.28\ {}^{+9}_{-6}\ {}^{+4}_{-5}$ |
| GSS [22] | 0.059 | $0.50(14)\ {}^{+7}_{-5}$ | $0.61(23)\ {}^{+9}_{-6}$ | $0.16(4)\ {}^{+22}_{-16}$ | $0.72(35)\ {}^{+10}_{-7}$ |

## 3. Semi-leptonic $B$ decays: heavy-to-light and heavy-to-heavy transitions

Recently, there has been much activity in studying the decays $B \to \pi\ell\overline{\nu}_\ell$, $\rho\ell\overline{\nu}_\ell$, which can be used to extract $V_{ub}$. The matrix elements for these decays are parametrised in terms of form factors, e.g. for $B \to \pi\ell\overline{\nu}_\ell$

$$\langle\pi|V_\mu|B\rangle = (p_B + p_\pi)_\mu\, f_+(q^2) - (p_B - p_\pi)_\mu\, f_-(q^2), \tag{9}$$

where $q = p_B - p_\pi$ is the momentum transfer. For the decay $B \to \rho\ell\overline{\nu}_\ell$, one has additional form factors $V$, $A_1$, $A_2$ and $A_3$. An important ingredient in the analysis of these decays is the observation that for infinite heavy quark mass, HQET predicts scaling laws for the form factors near maximum momentum transfer $q^2_{\rm max}$, i.e. at leading order in $1/M$ [15]

$$f^+(q^2_{\rm max}) \sim M^{1/2}, \quad V(q^2_{\rm max}) \sim M^{1/2}, \quad A_1(q^2_{\rm max}) \sim M^{-1/2}, \ldots \tag{10}$$

Due to the limitations imposed by the lattice spacing, simulations are not yet suited for a direct computation of form factors for $B \to \pi\ell\overline{\nu}_\ell$, $\rho\ell\overline{\nu}_\ell$. One rather obtains the form factors for typical lattice momenta $|\vec{p}| \leq 1.5\,\mathrm{GeV}/c$ and for heavy quark masses in the region of charm. Hence, the "generic" semi-leptonic heavy-to-light transition in current lattice simulations is $D \to K\ell\overline{\nu}_\ell$, and typical momentum transfers lie in the range $-0.8\,\mathrm{GeV}^2/c^2 \leq q^2 \leq 1.7\,\mathrm{GeV}^2/c^2$. Lattice results for form factors relevant for semi-leptonic $D$ decays can be found in a recent review [16].

In order to make predictions for $B \to \pi\ell\overline{\nu}_\ell$, $\rho\ell\overline{\nu}_\ell$, one needs to extrapolate the lattice form factors to the mass of the $b$ quark using the above scaling laws. Since the range of accessible lattice momenta is rather restricted, i.e. $|\vec{p}| \ll m_b$, one obtains the form factors in a narrow range of $q^2$ near $q^2_{\rm max}$. Therefore, in order to determine the $q^2$-behaviour of form factors for $B \to \pi\ell\overline{\nu}_\ell$, $\rho\ell\overline{\nu}_\ell$, or their values at $q^2 = 0$, one cannot avoid introducing a certain model dependence in the lattice results: assuming vector pole dominance is not reliable, since the accessible range of $q^2$ is rather narrow, and lattice data can presently not distinguish between different types of pole behaviour. In an alternative procedure, one first interpolates the lattice data to $q^2 = 0$ for quark masses around $m_{\rm charm}$. But in order to extrapolate the resulting form factors at $q^2$ to the mass of the $b$ quark, one needs to guess its leading scaling behaviour in the heavy mass, which, in contrast to eq. (10), cannot be obtained from HQET. Several methods have been applied, and Table 2 lists the results for form factors at $q^2 = 0$ from various groups. It has been noted that the model dependence could be avoided in the framework of light-cone sum rules [17], where one

Table 3: Lattice results for the slope parameter $\rho^2$ of the Isgur-Wise function, the applied parametrisation and, where quoted, estimates for $|V_{cb}|$ using measured decay rates. In the last column, we also list the form factor on which the estimate is based, or which method was used to formulate heavy quarks.

| Collab. | $a$ [fm] | $\rho^2_{u,d}$ | $\rho^2_s$ | Par. | $\lvert V_{cb}\rvert$ | |
|---|---|---|---|---|---|---|
| BSS [26] | 0.059 | | 1.21(26)(33) | lin. | 0.044(5)(7) | $h_+$ |
| UKQCD [25] | 0.069 | $0.9\ ^{+2}_{-3}\ ^{+4}_{-2}$ | $1.2\ ^{+2}_{-2}\ ^{+2}_{-1}$ | BSW | $0.037(1)(2)\ ^{+4}_{-1}$ | $h_+$ |
| UKQCD [27] | 0.069 | 1.1(5) | $1.2\ ^{+2}_{-3}$ | BSW | 0.037(3)(5) | $h_{A_1}$ |
| LANL [28] | 0.083 | 0.97(6) | | BSW | | $h_+$ |
| MO [29] | 0.170 | | 0.95 | quad. | | static |
| Ken [30] | 0.083 | 0.41(2) | | | | static |
| HM [31] | 0.083 | 0.70(17) | | lin. | | NRQCD |

obtains a leading scaling behaviour of $F(0) \sim M^{-3/2}$ for all form factors. This argument has so far not been directly applied in lattice simulations.

It has also been suggested[21] to use lattice form factors to calculate the differential decay rate for the *exclusive* decay $\overline{B}^0 \to \rho^+\ell^-\overline{\nu}_\ell$ beyond the region of charm production. One can then avoid the difficult determination of $F(q^2=0)$, and a model independent extraction of $|V_{ub}|$ is possible using experimental data for the differential decay rate $d\Gamma/dq^2$

$$|V_{ub}|^{-2}\frac{d\Gamma}{dq^2} \propto \left\{|H^+(q^2)|^2 + |H^-(q^2)|^2 + |H^0(q^2)|^2\right\} = \mathcal{A}^2\left(1+\mathcal{B}(q^2-q^2_{\rm max})\right), \qquad (11)$$

where $H^\pm(q^2)$ and $H^0(q^2)$ are combinations of the form factors $A_1(q^2)$, $A_2(q^2)$ and $V(q^2)$. The term $\mathcal{A}^2(1+\mathcal{B}(q^2-q^2_{\rm max}))$ parametrises long-distance hadronic dynamics. In fact, $\mathcal{A}^2$ plays the same rôle as the Isgur-Wise function in the case of heavy-to-heavy transitions. Here, lattice data can be used to determine the normalisation, whereas in the heavy-to-heavy case, the overall normalisation is provided by the HQ-symmetry. In a recent study[21], the authors obtain $\mathcal{A}^2 = 21\pm 3$ GeV$^2$ and $\mathcal{B} = (-8\ ^{+4}_{-6})\,10^{-2}$ GeV$^{-2}$.

Heavy-to-heavy transitions like $B\to D\ell\overline{\nu}_\ell$, $D^*\ell\overline{\nu}_\ell$ are also parametrised in terms of six form factors, $h_+, h_-, h_V, h_{A_1}, h_{A_2}$ and $h_{A_3}$, which are functions of $\omega$, the product of 4-velocities of the $B$ and $D$ mesons. HQ-symmetry relates the six form factors to one universal form factor, $\xi(\omega)$, called the Isgur-Wise function, which is normalised at zero recoil, $\xi(1)=1$. The form factor $h_+(\omega)$ is related to $\xi(\omega)$ via

$$h_+(\omega) = (1+\beta_+(\omega)+\gamma_+(\omega))\,\xi(\omega), \qquad (12)$$

where $\beta_+(\omega)$ parametrises radiative corrections between HQET and full QCD, and $\gamma_+(\omega)$ denotes (unknown) corrections in the inverse heavy quark mass.

On the lattice one typically obtains the form factors from a ratio of the relevant matrix elements at $\omega$ and at zero recoil ($\omega=1$). Thereby, some of the systematic effects cancel, most notably the normalisation of the axial and vector currents. In order to obtain an estimate for $\xi(\omega)$, the known radiative corrections can also be subtracted. Lattice data for the form factors $h_+$ and $h_{A_1}$ have been used to test the HQ-symmetry [24,25]. It has been found that $h_+(\omega)$ shows only a weak dependence on the heavy quark mass, and that

Table 4: Masses and mass splittings of heavy baryons from various collaborations.

| | | $h$ =charm | | $h$ =beauty | |
|---|---|---|---|---|---|
| | Collab. | Latt. [GeV] | Exp. [GeV] | Latt. [GeV] | Exp. [GeV] |
| $\Lambda_h$ | UKQCD [34] | $2.27\ ^{+4}_{-3}\ ^{+3}_{-3}$ | 2.285(1) | $5.64\ ^{+5}_{-5}\ ^{+3}_{-2}$ | 5.641(50) |
| | PCW [33] | | | 5.728±0.144±0.018 | |
| $\Sigma_h$ | UKQCD [34] | $2.46\ ^{+7}_{-3}\ ^{+5}_{-5}$ | 2.453(1) | $5.77\ ^{+6}_{-6}\ ^{+4}_{-4}$ | 5.814(60) |
| $\Sigma^*_h$ | UKQCD [34] | $2.44\ ^{+6}_{-4}\ ^{+4}_{-5}$ | 2.530(7) | $5.78\ ^{+5}_{-6}\ ^{+4}_{-3}$ | 5.870(60) |
| $\Xi_h$ | UKQCD [34] | $2.41\ ^{+3}_{-3}\ ^{+4}_{-4}$ | 2.468(4) | $5.76\ ^{+3}_{-5}\ ^{+4}_{-3}$ | |
| $\Omega_h$ | UKQCD [34] | $2.68\ ^{+5}_{-4}\ ^{+5}_{-6}$ | 2.704(20) | $5.99\ ^{+5}_{-5}\ ^{+5}_{-5}$ | |
| | Collab. | Latt. [MeV] | Exp. [MeV] | Latt. [MeV] | Exp. [MeV] |
| $\Lambda_h - P$ | UKQCD [34] | $408\ ^{+41}_{-31}\ ^{+34}_{-35}$ | 417(1) | $359\ ^{+55}_{-45}\ ^{+27}_{-26}$ | 362(50) |
| | ELC [35] | | | $720 \pm 160\ ^{+\ 0}_{-130}$ | |
| | UKQCD [12] | | | $420\ ^{+100}_{-\ 90}\ ^{+30}_{-30}$ | |
| | PCW [33] | $564 \pm 88 \pm 18$ | | $458 \pm 144 \pm 18$ | |
| $\Sigma_h - \Lambda_h$ | UKQCD [34] | $190\ ^{+50}_{-43}\ ^{+13}_{-13}$ | 169(2) | $157\ ^{+52}_{-64}\ ^{+11}_{-11}$ | 173(11) |
| $\Sigma^*_h - \Sigma_h$ | UKQCD [34] | $-17\ ^{+12}_{-31}\ ^{+3}_{-2}$ | 77(6) | $-6\ ^{+\ 4}_{-11}\ ^{+1}_{-1}$ | 56(16) |
| $\Xi^*_h - \Xi'_h$ | | $-20\ ^{+12}_{-24}\ ^{+2}_{-3}$ | 83 | $-7\ ^{+4}_{-8}\ ^{+1}_{-1}$ | |
| $\Omega^*_h - \Omega'_h$ | | $-23\ ^{+\ 6}_{-14}\ ^{+3}_{-2}$ | | $-8\ ^{+2}_{-5}\ ^{+1}_{-1}$ | |

the Isgur-Wise function extracted from both $h_+$ and $h_{A_1}$ is compatible within statistical errors. This may be interpreted as a manifestation of the HQ-spin-flavour symmetry.

In order to extract $|V_{cb}|$ from the experimentally measured decay rate for $B \to D^* \ell \overline{\nu}_\ell$ using lattice data, one first needs to parametrise $\xi(\omega)$. One particular parametrisation is

$$\xi_{\rm BSW}(\omega) = \tfrac{2}{\omega+1}\ \exp\left\{-(2\rho^2-1)\tfrac{\omega-1}{\omega+1}\right\}, \tag{13}$$

where $\rho^2$ is the slope of $\xi(\omega)$ at zero recoil. Other parametrisations, based on linear, quadratic and pole *ansätze*, have also been studied. Lattice data have so far not revealed any significant dependence on the chosen parametrisation[23,25].

Table 3 contains lattice results for the slope, either for massless spectator quarks ($\rho^2_{u,d}$), or in the case that the spectator quark is a strange quark ($\rho^2_s$). Also, the parametrisation and the estimates of $|V_{cb}|$ are given. Lattice results for the slope are consistent among the different collaborations, although the errors are still large. Recently, preliminary results [32] for semi-leptonic $\Lambda_b \to \Lambda_c$ decays have been shown, which will be valuable in further studies of the HQ-symmetry.

## 4. Heavy Baryon Spectroscopy

There has been increased experimental and theoretical activity in the study of baryons containing one heavy quark. Lattice calculations can make predictions for the masses of heavy baryons, which are extracted from the exponential fall-off of correlation functions of suitably chosen interpolating operators. The results from two recent studies [33,34] are listed

in Table 4, and one observes good agreement with experimental data. Mass splittings involving heavy baryons have also been studied by a number of groups, and the results are also given in Table 4. The agreement between experimental and lattice data for the $\Lambda-$ Pseudoscalar and $\Sigma - \Lambda$ splittings is rather good. However, the spin splittings, in particular for $\Sigma_c^* - \Sigma_c$, are definitely inconsistent with experiment. This is attributed to a convolution of lattice artefacts and the quenched approximation.

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